MENIIT

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JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 25-06-2022 (MORNING SHIFT) | TIME : (9.00 AM to 12.00 PM)

Duration 3 Hours | Max. Marks : 300

QUESTIONS & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. If $Z = \frac{A^2B^3}{C^4}$, then the relative error in Z will be:

(A) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$ (B) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$ (D) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

Ans. (B)

Sol. $Z = \frac{A^2B^3}{C^4}$

$\ell n(Z) = 2\ell nA + 3\ell nB - 4\ell nC$

$\left(\frac{dZ}{Z}\right) = 2\frac{\Delta A}{A} + 3\frac{\Delta B}{B} + 4\frac{\Delta C}{C}$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}|$ =non-zero constant. Which of the following expression is true for $\vec{A}$?

(A) $\vec{A}\square\vec{A} = 0$ (B) $\vec{A} \times \vec{A} < 0$ (C) $\vec{A} \times \vec{A} = 0$ (D) $\vec{A} \times \vec{A} > 0$

Ans. (C)

Sol. $\vec{A} \times \vec{A} = |A|$

$|A|\sin\theta = 0$

3. Which of the following relations is true for two unit vector $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

(A) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}|\tan\frac{\theta}{2}$ (B) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}|\tan\frac{\theta}{2}$

(C) $|\hat{A} + \hat{B}| = |\hat{A} - \hat{B}|\cos\frac{\theta}{2}$ (D) $|\hat{A} - \hat{B}| = |\hat{A} + \hat{B}|\cos\frac{\theta}{2}$

Ans. **(B)**

Sol $|\hat{A} + \hat{B}| = \sqrt{(1)^2 + (1)^2 + 2(1)(1)\cos\theta} = \sqrt{2(1+\cos\theta)}$

$= \sqrt{2\left(2\cos^2\frac{\theta}{2}\right)} = 2\cos\frac{\theta}{2}$

$|\hat{A} - \hat{B}| = \sqrt{(1)^2 + (1)^2 - 2(1)(1)\cos\theta} = \sqrt{2(1-\cos\theta)}$

$\sqrt{2\left(2\sin^2\frac{\theta}{2}\right)} = 2\sin\frac{\theta}{2}$

$$\frac{|\hat{A}-\hat{B}|}{|\hat{A}+\hat{B}|}=\frac{2\sin\frac{\theta}{2}}{2\sin\frac{\theta}{2}}=\tan\left(\frac{\theta}{2}\right)$$

$$=|\hat{A}-\hat{B}|=\tan\left(\frac{\theta}{2}\right)|\hat{A}+\hat{B}|$$

4. If force $\vec{F}=3\hat{i}+4\hat{j}-2\hat{k}$ acts on a particle having position vector $2\hat{i}+\hat{j}-2\hat{k}$ then, the torque about the origin will be:

(A) $3\hat{i}+4\hat{j}-2\hat{k}$ (B) $-10\hat{i}+10\hat{j}+5\hat{k}$ (C) $10\hat{i}+5\hat{j}-10\hat{k}$ (D) $10\hat{i}+\hat{j}-5\hat{k}$

Ans. **(B)**

Sol. $\vec{\tau}=\vec{r}\times\vec{F}=(2\hat{i}+1\hat{j}+2\hat{k})\times(3\hat{i}+4\hat{j}-2\hat{k})$

$$\vec{\tau}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&1&2\\3&4&-2\end{vmatrix}=(\hat{i})(-2-8)-\hat{j}(-4-6)+\hat{k}(8-3)$$

$$-10\hat{i}+10\hat{j}+5\hat{k}$$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be: (Given g = acceleration due to gravity at the surface of earth).

(A) g/2 (B) g/4 (C) g/3 (D) g/9

Ans. **(D)**

Sol. $g_{out}=\frac{GM}{r^2}=\frac{Gm}{(R+R)^2}=\frac{1GM}{9R^2}$

$$g_{out}=\frac{g_s}{9}$$

6. The terminal velocity (υ_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as :

(A) $r^{1/2}$ (B) r (C) r^2 (D) r^3

Ans. **(C)**

Sol. $v_t=\frac{2}{9}\frac{(\rho-\rho_i)gr^2}{4}\Rightarrow v_t\propto r^2$

7. The relation between root mean square speed (υ_{rms}) and most probable speed (υ_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be:

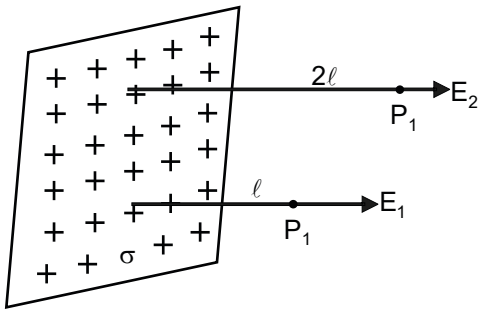
(A) $\upsilon_{rms}=\sqrt{\frac{2}{3}}\upsilon_p$ (B) $\upsilon_{rms}=\sqrt{\frac{3}{2}}\upsilon_p$ (C) $\upsilon_{rms}=\upsilon_p$ (D) $\upsilon_{rms}=\sqrt{\frac{1}{3}}\upsilon_p$

Ans. **(B)**

Sol. $v_{rms}=\sqrt{\frac{3RT}{M}}$

$v_{mp} = \sqrt{\frac{2RT}{M}}$; $\frac{V_{rms}}{V_{mp}} = \sqrt{\frac{3}{2}}$

8. In the figure a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance ℓ and 2ℓ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 at E_2 at P_1 and P_2 respectively are:



(A) $E_1 = \sigma/\varepsilon_o$, $E_2 = \sigma/2\varepsilon_o$ (B) $E_1 = 2\sigma/\varepsilon_o$, $E_2 = \sigma/\varepsilon_o$

(C) $E_1 = E_2 = \sigma/2\varepsilon_o$ (D) $E_1 = E_2 = \sigma/\varepsilon_o$

Ans. **(C)**

Sol. Electric field due to a large non-conducting sheet

$E = \frac{\sigma}{2\varepsilon_o}$ and it is uniform width distance from the sheet

9. Match **List - I** with **List – II.**

List - I	**List - II**
(A) AC generator	(I) Detects the presence of current in the circuit
(B) Galvanometer	(II) Converts mechanical energy into electrical energy
(C) Transformer	(III) Works on the principal of resonance in AC circuit
(D) Metal detector	(IV) Changes an alternating voltage for smaller or greater value

Choose the correct answer from the options given blow:

(A) (A) - (II), (B) - (I), (C) – (IV), (D) – (III) (B) (A) - (II), (B) - (I), (C) – (III), (D) – (IV)

(C) (A) - (III), (B) - (IV), (C) – (II), (D) – (I) (D) (A) - (III), (B) - (I), (C) – (II), (D) – (IV)

Ans. **(A)**

Sol. Transformer is used in charging the amplitude of alternating voltage. A metal detector works on the principal of resonance

10. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r(r > R) from its center will be:

(A) $B \propto r^2$ (B) $B \propto r$ (C) $B \propto \frac{1}{r^2}$ (D) $B \propto \frac{1}{r}$

Ans. **(B)**

Sol. $B_{in} = \left(\frac{\mu_0 i}{2\pi R^2}\right) n \Rightarrow B_{in} \propto n$

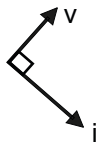
11. If wattless current flows in the AC circuit, then the circuit is:

(A) Purely Resistive circuit (B) Purely Inductive circuit

(C) LCR series circuit (D) RC series circuit only

Ans. **(B)**

Sol. If the entire current passing through a wire is wattles then the phase difference between I and v should be $\frac{\pi}{2}$, so the circuit should be either purely inductive or purely capacitive.



12. The electric field in an electromagnetic wave is given by E = 56.5 ω(t-x/c) NC^{-1}. Find the intensity of the wave if it is propagating along x-axis in the free space.

(Given $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$)

(A) 5.65 Wm^{-2} (B) 4.24 Wm^{-2} (C) 1.9×10^{-7} Wm^{-2} (D) 56.5 Wm^{-2}

Ans. **(B)**

Sol. Average electrical energy density = $\frac{1}{2}\varepsilon_0 E_{rms}^2 = \frac{1}{4}\varepsilon_0 E_0^2$

$$\frac{dE}{Adx} = \frac{1}{4}\varepsilon_0 E_0^2$$

dividing get $\frac{dx}{dt} = \frac{I}{\frac{1}{2}\varepsilon_0 E_0^2}$ where $\frac{dx}{dt} = C$

$$C = \frac{I}{\frac{1}{2}\varepsilon_0 E_0^2} \Rightarrow I = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)(C)$$

$I = \frac{1}{2}(8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8) = 4.24 \times W/m^2$

13. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point and π at point Q. Then the difference between the resultant intensities at P and Q will be:

(A) 2 I (B) 6 I (C) 5 I (D) 7 I

Ans. **(B)**

Sol. $I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\frac{\pi}{2}$

$I_P = I_1 + I_2 = 10I$

$I_Q = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\pi$

$= 9I + I + 2 \times 3I(-1) = 4I$

$\frac{I_P}{I_Q} = \frac{10}{4} = \frac{5}{2}$

14. A light wave travelling linearly in a medium of dielectric constant 4, incidents on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be:

(Given : relative permeability of medium $\mu_r = 1$)

(A) 10° (B) 20° (C) 30° (D) 60°

Ans. **(D)**

Sol. $n = \sqrt{\mu_r \varepsilon_r}$

$n_{ree} = \frac{n_D}{n_r} = \frac{\sqrt{4 \times 1}}{\sqrt{1 \times 1}} = 2$

$\theta > c$

$\sin\theta > \sin c$

$\sin\theta > \frac{1}{n_{ree}}$

$\sin\theta > ½$

$\theta > 30°$

15. Given below are two statements:

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the **correct answer** from the option given below:

(A) Both **Statement I** and **Statement II** are true.

(B) Both **Statement I** and **Statement II** are false.

(C) **Statement I** is true but **Statement II** is false.

(D) **Statement I** is false but **Statement II** is true.

Ans. **(A)**

Sol. In Davissor – Germer experiment, the high speed electrons are incident on Ni crystal and they get diffracted just like a wave. So wave nature of electron was experimentally observed in this experiment.

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be:

(A) 1 : 1 (B) 1 : 2 (C) 4 : 1 (D) 2 : 1

Ans. **(D)**

Sol. $v = (2.19 \times 10^6 \text{ m/sec.}) \frac{z}{n} \Rightarrow v \propto \frac{z}{n}$

$v_1 \propto \frac{2}{3}, v_2 \propto \frac{1}{3}$

$\frac{v_1}{v_2} = \frac{2}{1}$

17. The photodiode is used to detect the optical signals. These diodes are preferably operated in reverse biased mode because :

(A) fractional change in majority carries produce higher forward bias current

(B) fractional change in majority carries produce higher reverse bias current

(C) fractional change in minority carries produce higher forward bias current

(D) fractional change in minority carries produce higher reverse bias current

Ans. **(D)**

Sol. In photo–diode, the diode is connected in reverse bias, so initially the current will be very small. If light is incident on the diode, its photons produce more minority charge carriers, which are called photo generated charge carriers, due to which the reverse current increases. In reverse bias, the current is already very small, So fractional charge in the current due to the light will be large.

18. A signal of 100 THz frequency can be transmitted with maximum efficiency by :

(A) Coaxial cable (B) Optical fiber

(C) Twisted pair of copper wires (D) Water

Ans. **(B)**

Sol. Such a large frequency signals (100THZ) can be transmitted by optical fiber.

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum c = 3×10^8 ms^{-1})

(A) 1.303 (B) 1.318 (C) 1.13 (D) 0.12

Ans. (C)

Sol. $V_A = \frac{C_0}{n_A}, V_B = \frac{C_0}{n_B}$

$V_A - V_B = C_0\left(\frac{1}{n_A} - \frac{1}{n_B}\right) = 2.6 \times 10^7$

$(3 \times 10^8)\left(\frac{1}{1.47} - \frac{1}{n_B}\right) = 2.6 \times 10^7$

$n_B = 1.67 \Rightarrow \frac{n_B}{n_A} = \frac{1.67}{1.47} = 1.13$

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) a galvanometer, Students took the observations for $\frac{1}{3}$ deflection in the galvanonmeter. Which of the below is **true** for measuring value of G?

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

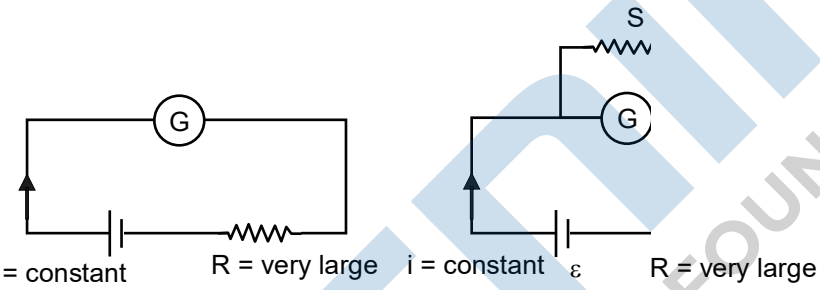
(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of stunt resistance(s).

(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three time the value of stunt resistance(s).

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the stunt resistance(s).

Ans. **(B)**

Sol.



If a large resistance R is connected with a battery, then current will be approximately $\frac{\varepsilon}{R}$, and t will remain almost constant.

In 1st case, only (G) is connected $\Rightarrow (i_g)_1 = i$

In 2ed case, $(i_g)(R_g) = (I - i_g)(S) \Rightarrow (i_g)_2 = \left(\frac{S}{R_g + S}\right) i$

If $(i_g)_2 = \frac{1}{3}(I_g), \Rightarrow \left(\frac{S}{R_g + S}\right) i = \frac{1}{3} i$

$R_g = 2(S)$

Numeric Value Type

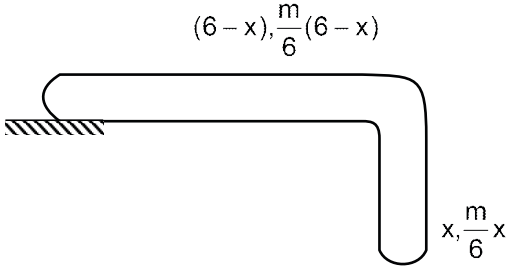
This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. A uniform chain of 6m length is placed on a table such that a part of length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _____m.

Ans. **2**

Sol.

To prevent sliding $\left(\frac{m}{6}x\right)g \le \left(\frac{m}{6}(6-x)g\right)\mu_s$



$x \le 2 \quad \Rightarrow \quad x_{max} = 2$

22. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30cm when its speed is halved. The spring constant of the spring will be _____ Nm^{-1}

Ans. **600**

Sol. $\frac{1}{2}mV^2 = \frac{1}{2}m\left(\frac{V^2}{4}\right) + \frac{1}{2}k(0.3)^2$

$\frac{1}{2}m\frac{3}{4}V^2 = \frac{1}{2}k(0.9)$

K = 600 N/m

23. The velocity of upper layer of water in a river is 36kmh^{-1}. She stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is _____ m. (Co-efficient of viscosity of water is 10^{-2} Pa.s)

Ans. **100**

Sol. $F_v = \eta A \frac{\Delta v}{\Delta y}$

Shear stress = $\frac{F_v}{A} = \eta\frac{\Delta v}{\Delta y}$

$10^{-3} = (10^{-2})\frac{10-0}{h} \Rightarrow h = 100m$

24. A steam engine intakes 50g of steam at 100°C per minute and cools is down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is _____ $\times 10^3$ cal.

(Given: specific heat capacity of water: 1 cal g^{-1} $°C^{-1}$)

Ans. **31**

Sol. 50 × 540 cal + 50(1) (80) cal

= 50(540 + 80)

= 50 × 620 = 31000 cal

= 31 kcal

= 31 × 4.2

$= 130.2 \times 10^3$ J

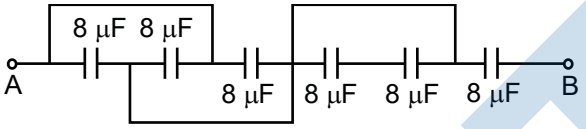
25. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20cm. The length of the open organ pipe is - _____ cm.

Ans. **80**

Sol. $\frac{2V}{2\ell_1} = \frac{V}{4\ell_2}$ $\ell_1 = 4\ \ell_2 = 4 \times 21$

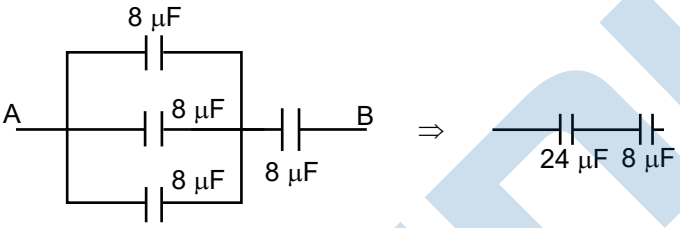
$\ell_1 = 80$cm

26. The equivalent capacitance between points A and B in below shown figure will be μF.



Ans. **6**

Sol.



$\Rightarrow$

$C_{AB} = \frac{24 \times 8}{24 + 8} = 6\mu F$

27. A resistor develops 300 J pf thermal energy in 15s, when a current of 2 A is passed through it. If the current increases to 3 A, the energy developed in 10 s is _____ J.

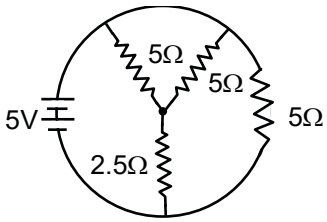
Ans. **450**

Sol. H = i2 Rt

$\frac{H_2}{H_1} = \frac{i_2^2 t_2}{i_1^2 t_1} = \frac{3^2 \times 10}{2^2 \times 15}$

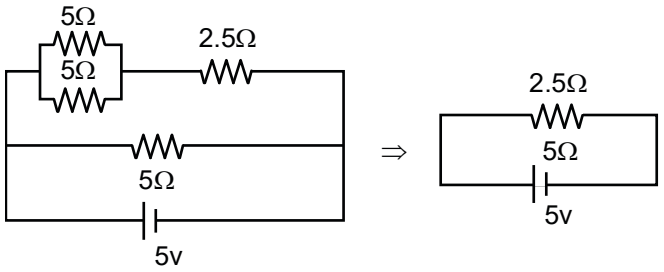
H2 = 450 J

28. The total current supplied to the circuit as shown in figure by the 5 V battery is _____ A.



Ans. **2**

Sol.



$$i = \frac{5}{2.5} = 2A$$

29. The current in a coil of self inductance 2.0 H is increasing according to 1 = 2 sin(t^2) A. The amount of energy spent during the period when current changes from 0 to 2 A is _____ J.

Ans. **4**

Sol. $\Delta E = \frac{1}{2}Li_f^2 - \frac{1}{2}Li^2$

$$\Delta E = \frac{1}{2}(2)\,(2^2 - 0^2) = 4J$$

30. A force on an object of mass 100g is $(10\hat{i} + 5\hat{j})$ N. The position of that object at t = 2s is $(a\hat{i} + b\hat{j})$ m after starting from rest. The value of $\frac{a}{b}$ will be _____.

Ans. **2**

Sol. $\vec{a} = \frac{f}{m} = 20\hat{i} + 10\hat{j} m/s$

$$s = \frac{1}{2} \times 20 \times 2^2 \hat{i} + \frac{1}{2} \times 10 \times 2^2 \hat{j} m$$

$$\frac{a}{b} = \frac{2}{1}$$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron?

(A) NO (B) N_2 (C) O_2 (D) C_2

(E) B_2

Choose the most appropriate answer from the options given below:

(A) (A), (B), (C) only (B) (B), (C), (E) only (C) (A), (C) only (D) (D) only

Ans. (C)

Sol.

Species	B_2	NO	N_2	O_2	C_2
Bond order	1	2.5	3	2	2
Species	B_2^+	NO^+	N_2^+	O_2^+	C_2^+
Bond order	0.5	3	2.5	2.5	1.5

2. Incorrect statement for Tyndall effect is:

(A) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

(B) The diameter of the dispersed particles is much smaller then the wavelength of the light used.

(C) During projection of movies in the cinemas hall, Tyndall effect is noticed.

(D) It used to distinguish a true solution from a colloidal solution.

Ans. **(B)**

Sol. *The diameter of the dispersed particles is not much smaller than the wavelength of the light used

*The intensity of scattered light depends on the difference between the refractive indice of the D.P and D.M., In lyophobic colloids, this difference is appreciable and therefore the tyndal effect is quite well defined but in lyophilic sols the difference is very small and the tyndall effect is very weak.

So, to show Tyndall effect the refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

3. The pair, in which ions are isoelectronic with Al^{3+} is :

(A) Br^- and Be^{2+} (B) Cl^- and Li^+ (C) S^{2-} and K^+ (D) O^{2-} and Mg^{2+}

Ans. **(D)**

Sol.

Species	Al^{3+}	Mg^{2+}	Be^{2+}	O^{2-}	Br^-	Cl^-	Li^+	S^{2-}	K^+
No. of e^-	10	10	2	10	36	18	2	18	18

So Mg^{2+} & O^{2-} are isoelectronic with Al^{3+}

4. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are:

(A) $[Au(CN)_4]^-$ and $[Zn(CN)^2(OH)_2]^{2-}$ (B) $[Au(CN)_2]^-$ and $[Zn(OH)_4]^{2-}$

(C) $[Au(CN)_2]^-$ and $[Zn(CH)_4]^{2-}$ (D) $[Au(CN)_4]^{2-}$ and $[Zn(CH)_6]^{4-}$

Ans **(C)**

Sol. $4Au(s) + 8CN^-(aq) + 2H2O(aq) + O2(g) \longrightarrow 4[Au(CN)2]^-(aq) + 4OH-(aq)$

$2[Au(CN)2]^-(aq) + Zn(s) \longrightarrow [Zn(CN)4]^{2-}(aq) + 2\,Au(s)$

5. Number of electron deficient molecules among the following PH3, B_2H_6, CCl_4, NH_3, LiH and BCl_3 is

(A) 0 (B) 1 (C) 2 (D) 3

Ans **(C)**

Sol. Electron deficient among the given compounds: B2H6, BCl3

6. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution?

(A) Be^{2+} (B) Mg^{2+} (C) Ca^{2+} (D) Sr^{2+}

Ans (D)

Sol. Smaller the size of ion greater is it' s hydration & greater is it's hydrated radii & smaller is ionic mobility.

So order of ionic mobility: $Be^{2+} < Mg^{2+} < Ca^{2+} < Sr^{2+} < Ba^{2+}$

7. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of :

(A) $[Ag(NH_3)_4]Cl_2$ (B) $[Ag(Cl)_2(NH_3)_2]$ (C) $[Ag(NH_3)_2]Cl$ (D) $[Ag(NH_3)Cl]Cl$

Ans. **(C)**

Sol. $AgCl\downarrow + 2NH_4OH \rightarrow [Ag(NH_3)_2]Cl + 2H_2O$

8. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

(A) It will not prefer to undergo redox reactions.

(B) It will prefer to gain electron and act as an oxidizing agent

(C) It will prefer to give away an electron and behave as reducing agent

(D) it acts as both, oxidizing and reducing agent.

Ans **(B)**

Sol. Formation of Ce^{IV} is favoured by it' s noble gas configuration but it is strong oxidant reverting to the +3 state. The E° value for Ce^{4+}/Ce^{3+} is $E^0_{Ce^{4+}} / E^0_{Ce^{3+}} = 1.74V$ is favourable for its oxidising nature.

9. Among the following, which is the strongest oxidizing agent?

(A) Mn^{3+} (B) Fe^{3+} (C) Ti^{3+} (D) Cr^{3+}

Ans (A)

Sol. Oxidising agent gets reduced.

Due to much large third ionization energy of Mn(required for changing d^5 to d^4) is responsible for strong oxidising agent property of Mn^{3+}

Most stable oxidation state of

Ti $\rightarrow$ + 4; Cr $\rightarrow$ +3; Fe $\rightarrow$ +2; Mn $\rightarrow$ + 2 (acidic medium), 4(basic medium)

10. The eutrophication of water body result in :

(A) Loss of Biodiversity. (B) breakdown of organic matter.

(C) increase in biodiversity. (D) decrease in BOD.

Ans (A)

Sol. This process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.

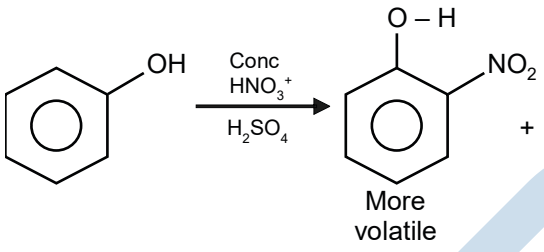
11. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most efficient for large scale separation?

(A) Chromatographic separation (B) Fraction Crystallisation

(C) Steam distillation (D) sublimation

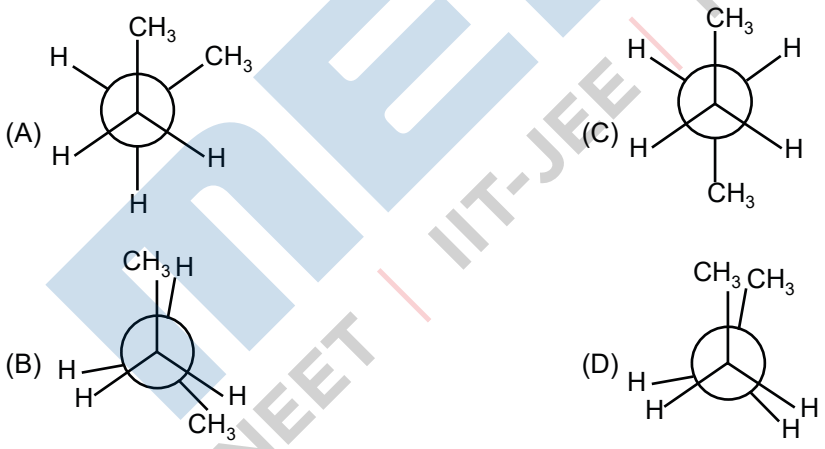
Ans **(C)**

Sol.



Mixture of ortho &para nitro phenol is separated by steam distillation as ortho isomer is steam volatile.

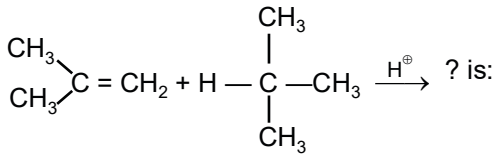
12. In the following structures, which one is having staggered conformation with maximum dihedral angle?

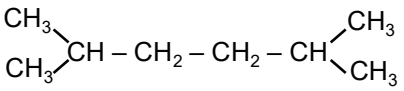


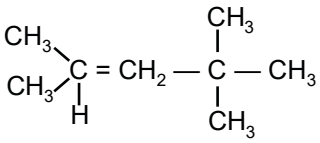
Ans **(C)**

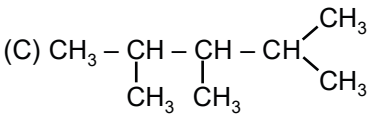
Sol. Anti-conformation of butane along C_2–C_3 bond has maximum dihedral angle i. e. 180º.

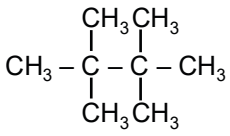
13. The product formed in the following reaction.



(A) 

(B) 

(C) 

(D) 

Ans. **(B)**

Sol. $(CH_3)_2C = CH_2 \xrightarrow{H^+} (CH_3)_3C^+ + (CH_3)_2C = CH_2 \longrightarrow (CH_3)_3C - CH_2 - C^+(CH_3)_2$

$\longrightarrow (CH_3)_3C - CH_2 - CH(CH_3)_2$

14. The IUPAC name of ethylidene chloride is:

(A) 1-Chloroethene

(B) 1-Chloroethene

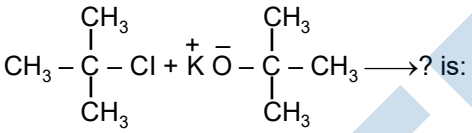
(C) 1,2-Dichloroethene

(D) 1,1-Dichloroethene

Ans. (D)

Sol. $CH_3 - CH - Cl_2$ (1,1-Dichloroethane)

15. The major product in the reaction



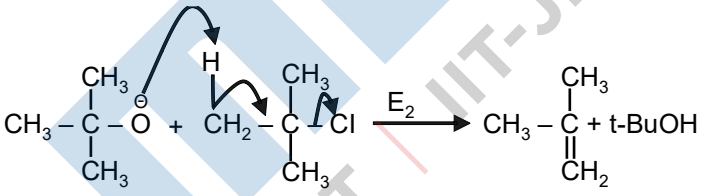
(A) *t*-Butyl ethyl ether

(B) 2,2-Dimethyl butane

(C) 2-Merhyl pent-1-ene

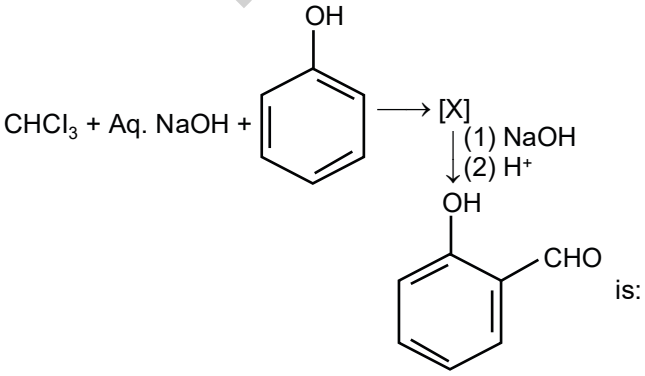
(D) 2-Methyl prop-1-ene

Ans **(D)**

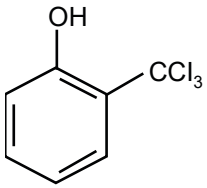
Sol. 

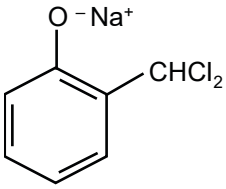
t-butoxide ion carryout Elimination reaction at 3º alkyl halide

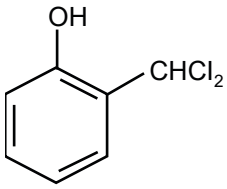
16. The intermediate X, in the reaction:

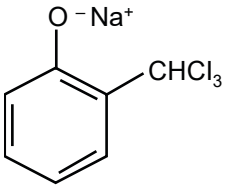


is:

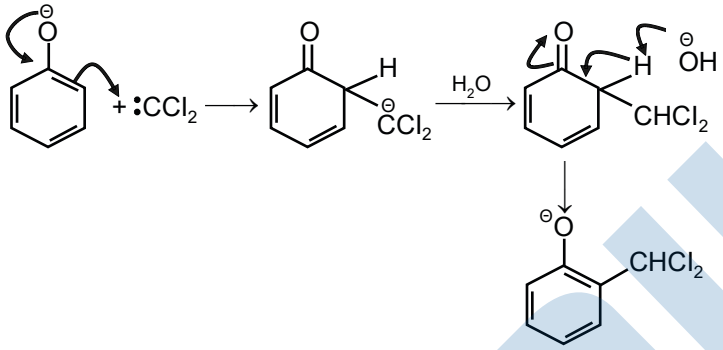
(A) 

(C) 

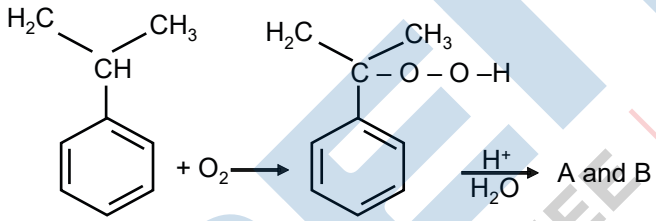
(B) 

(D) 

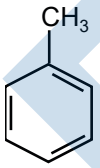
Ans **(C)**

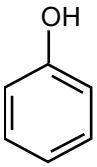
Sol. 

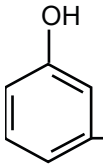
17. In the following reaction :

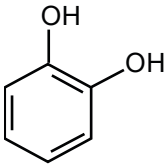


The compounds A and B respectively are :

(A)  , CH_3COOH

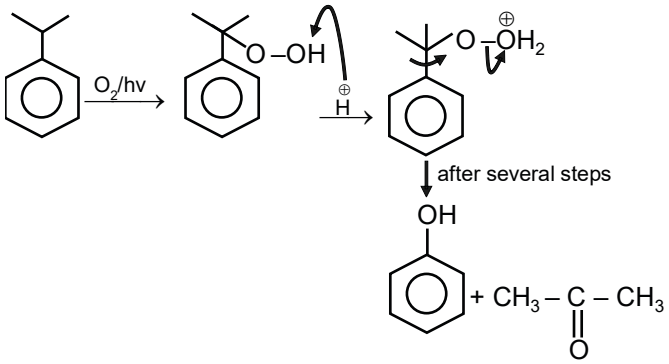
(C)  , CH_3COCH

(B)  , CH_3COCH

(D)  , CH_3COCH_3

Ans. **(C)**

Sol.



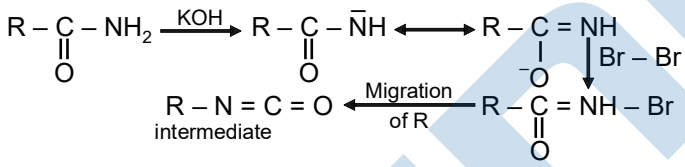
Cumene hydroperoxide undergoes rearrangement to give phenol and acetone.

18. The reaction of $R-\underset{\underset{O}{\|}}{C}-NH_2$ with bromine and KOH gives RHN_2 as the product. Which one of the following is the intermediate product formed in the reaction?

(A) $R-\underset{\underset{O}{\|}}{C}-NH-Br$ (B) $R-NH-Br$ (C) $R-N=C=O$ (D) $R-\underset{\underset{O}{\|}}{C}-NBr_2$

Ans (C)

Sol.



19. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes, because:

(A) Soap particles remain floating in water as ions.

(B) the hydrophobic part of soap is not able to take away grease.

(C) the micelles are not formed due to concentration of soap, below its CMC value.

(D) colloidal structure of soap in water is completely disturbed.

Ans **(C)**

Sol. Micelle or associate colloid formation occurs above a certain conc. Known as CMC.

20. Which one of the following is an example of artificial sweetner ?

(A) Bithional (B) Alitame (C) Salvarsan (D) Lactose

Ans. **(B)**

Sol. Alitame is an aspartic acid containing dipeptide artificial sweetener.

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. The number of N atoms in 681 g of $C_7H_5N_3O_6$ is $x \times 10^{21}$. The value of x is _____.

($N_A = 6.02 \times 10^{23}$ mol^{-1})

Ans **(5418)**

Sol. Moles of compound ($C_6H_5N_3O_6$) = $\frac{645}{215}$ = 3 mol

moles of Nitrogen = 9 mole

No. of atoms of Nitrogen = $9 \times 6.02 \times 10^{23}$

$= 54.18 \times 10^{23}$

$= 5418 \times 10^{21}$

22. The distance between Na^+ and Cl^- ions in NaCl of density 43.1 g cm^{-3} is _____ $\times 10^{-10}$ m.

Ans (1)

Sol. For NaCl Z = 4 & M = 58.5 gram

$$d = \frac{z \times m}{N_A \times \text{Volume}}$$

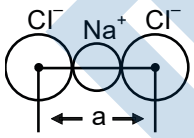
$$43.1 = \frac{4 \times 58.5}{6 \times 10^{23} \times [a]^3}$$

$$a^3 = \frac{4 \times 58.5}{6 \times 43.1} \times 10^{-23}$$

$= 0.9 \times 10^{-23}$

$= 9 \times 10^{-24}$

$a = 2.08 \times 10^{-8}$ cm



$$d_{Na^+ + Cl^-} = \frac{a}{2} = \frac{2.08 \times 10^{-10}}{2} m$$

23. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is $x \times 10^{-8}$ m. The value of x is _____.

(Given : Energy of the electron in the first shell of the hydrogen atom is -2.2×10^{-18} J ; h = 6.63×10^{-34} Js and c = 3×10^8 ms^{-1})

Ans **(4)**

Sol. Electronic configuration of Li = $1s^2\ 2s^1$

$(E_{Li})_{n=2} = (E_H)\frac{Z^2}{n^2} = -2.2 \times 10^{-18} \times \frac{9}{4}$

$E = \left(\frac{hc}{\lambda}\right) = 2.2 \times 10^{-18} \times \frac{9}{4}$

$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{\lambda} = 2.2 \times 10^{-18} \times \frac{9}{4}$

$\lambda = 4 \times 10^{-8}$ m

24. The standard entropy change for the reaction

$4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ is -550 J K^{-1} at 298 K.

[Given : The standard enthalpy change for the reaction is –165 kJ mol^{-1}]. The temperature in k at which the reaction attains equilibrium is _____.

Ans (300)

$4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ $\quad \Delta S_r = -550$ J/K

$\Delta S_r = -165$ kJ/mole

At equilibrium $\Delta G = 0$

$T = \left(\frac{\Delta H}{\Delta S}\right)$ at equilibrium

$= \frac{-165 \times 10^3}{-550} = 300$ K

25. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4. 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is _____$\times 10^3$ m mols.

Ans. **(20)**

Sol. NTA Answer is 15, but Zigyan answer is (20)

Initial moles of H_2SO_4 (in/Lit.) = 0.02

In 50% solution moles of H_2SO_4 = 0.01

Added moles of H_2SO_4 = 0.01

Total moles of H_2SO_4 in resulting solution = 0.02

$= 20 \times 10^{-3}$ moles

= 20 millimoles

Ans 20

26. The standard free energy change ($\Delta G°$) for 50% dissociation of N_2O_4 into NO_4 at 27°C and 1 atm pressure is – x J mol^{-1}. The value of x is _____.

[Given: R = 8.31 J K^{-1} mol^{-1}, log 1.33 = 0.1239 ln 10 = 2.3]

Ans. **(710)**

Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$

$1-\alpha \qquad\qquad 2\alpha$

$$k_p = \frac{4\alpha 2p}{1-\alpha^2} = \frac{4\times(0.5)\times 1}{1-(0.5)^2} = \frac{1}{0.75}$$

$$Kp = k_p = \frac{4}{3}$$

$\Delta G^o = -2.3\ RT \log kp$

$= -2.3 \times 8.31 \times 300 \times \log(1.33) = 710.4\ J\ mol^{-1} \approx 710\ J\ mol^{-1}$

27. In a cell, the following reactions take place

$Fe^{2+} \rightarrow Fe^{3+} + e^- \qquad E^\circ_{Fe^{3+}/Fe^{2+}} = 0.77\ V$

$2I^- \rightarrow I_2 + 2e^- \qquad E^\circ_{I_2/I^-} = 0.54\ V$

The standard electron potential for the spontaneous reaction in the cell is $x \times 10^{-2}$ V 298 K.

The value of x is _____.

Ans. (23)

Sol. $E^0_{Cell} = \left(E^0_{RP}\right)_C - \left(E^0_{RP}\right)_A$

$= 0.77 - 0.54$

$= 0.23V$

$= 23 \times 10^{-2}\ V$

Ans 23

28. For a given chemical reaction

$\gamma_1 A + \gamma_2 B \rightarrow \gamma_3 C + \gamma_4 D$

Concentration of C changes from 10 mmol dm^{-3} to 20 mmol dm^{-3} in 10 seconds. Rate of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be 9 mmol $dm^{-3}\ s^{-1}$. Therefore the rate of reaction is _____ mmol $dm^{-3}\ s^{-1}$.

Ans **(1)**

Sol. $\frac{d[c]}{dt} = \left(\frac{20-10}{10}\right) = 1$ milimole/dm^3 sec

$\gamma_1 A + \gamma_2 B \rightarrow \gamma_3 C + \gamma_4 D$

$$\frac{d[D]}{dt} = 1.5\left(-\frac{d}{dt}[B]\right) = 9 \text{ milimole/dm}^3 \text{ sec}$$

$$\left(-\frac{d}{dt}[B]\right) = 2\left(-\frac{d}{dt}[A]\right)$$

$$\text{Rate} = \frac{1}{\gamma_1}\frac{d[A]}{dt} = -\frac{1}{\gamma_2}\frac{d[B]}{dt} = \frac{1}{\gamma_3}\frac{d[C]}{dt} = \frac{1}{\gamma_4}\frac{d[D]}{dt}$$

(i) $\frac{d[D]}{dt} = -\frac{\gamma_4}{\gamma_2}\left[-\frac{d[B]}{dt}\right] = 1.5\left(-\frac{d}{dt}[B]\right)$

$\frac{\gamma_4}{\gamma_2} = 1.5$

(ii) $\frac{\gamma_2}{\gamma_1}\left[-\frac{d[A]}{dt}\right] = \left(-\frac{d}{dt}[B]\right) = 2\left(-\frac{d}{dt}[A]\right)$

$\frac{\gamma_2}{\gamma_1} = 2$

(iii) $\frac{d[C]}{dt} = \frac{\gamma_3}{\gamma_4}\left[-\frac{d[D]}{dt}\right]$

$1 = \frac{\gamma_3}{\gamma_4} \times 9$

$\frac{\gamma_4}{\gamma_3} = 9$

29. If $[Cu(H_2O)_4]^{2+}$ absorbs a light of wavelength 600 mm for d-d transition, then the value of octahedral crystal field splitting energy for $[Cu(H_2O)_6]^{2+}$ will be _____ $\times 10^{-21}$ J.

[Nearest integer]

(Given : h = 6.63×10^{-34} Js and c = 3.08×10^{8} ms^{-1})

Ans **(745)**

Sol. $\Delta_t = \frac{hc}{\lambda} = \left(\frac{6.63 \times 10^{-34} \times 3.08 \times 10^{8}}{600 \times 10^{-9}}\right) = 340.34 \times 10^{-21}$ J

$\Delta_o = \frac{9}{4}\Delta_t = \frac{9}{4} \times 340.34 \times 10^{-21} = 765.765 \times 10^{-21} \approx 766 \times 10^{21}$ J

30. Number of germs of bromine that will completely react with 5.0 g of pent-1-ene is_____ $\times 10^{-2}$ g. (Atomic mass of Br = 80 g/mol) [Nearest Integer]

Ans (1142)

Sol. NTA Answer is 1136, Zigyan answer is 1142.

Moles of Pent-1-ene = $\frac{5}{70}$

Moles of Br_2 = $\frac{5}{70}$

Weight of Br_2 = $\frac{5}{70} \times 160$

= 11.42

= 1142×10^{-2} g.

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line Passing through the canter of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is :

(A) $(x-1)^2 + (y-2)^2 = 4$ (B) $(x+1)^2 + (y-2)^2 = 4$

(C) $(x-1)^2 + (y+2)^2 = 16$ (D) $(x-1)^2 + (y-2)^2 = 16$

Ans. (C)

2. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to :

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Ans. (C)

Sol. $\int_0^{\pi} \frac{e^{\cos x}\sin x}{\left(e^{\cos x}+\frac{1}{e^{\cos x}}\right)\left(1+\cos^2 x\right)}dx$

By using P – VI

$$\int_0^{\frac{\pi}{2}} \left\{ \frac{e^{\cos x}\sin x}{\left(e^{\cos x}+\frac{1}{e^{\cos x}}\right)\left(1+\cos^2 x\right)} + \frac{\frac{1}{e^{\cos x}}\sin x}{\left(\frac{1}{e^{\cos x}}+e^{\cos x}\right)} \right\} dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$$

$\cos x = t$

$-\sin x\, dx = dt$

X	0	π/2
t	1	0

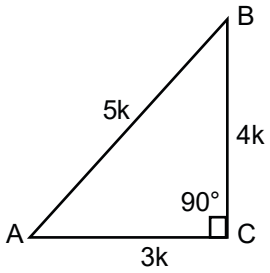
$$\int_1^0 \frac{-dt}{1+t^2} = \left(\cot^{-1} t\right)_1^0 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

3. Let a, n and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$ If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, Then the value of $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$ is equal to :

(A) $\frac{5}{2}$ (B) 2 (C) $\frac{3}{2}$ (D) 1

Ans. (A)

Sol.



a + b = 7 k

b + c = 8 k

a + c = 9 k

2(a + b + c) = 24 k

a + b + c =12 k

a = 4k, b = 3k, c= 5k (triangle is right angle at C)

$$2R = \frac{C}{\sin C} = \frac{5k}{\sin(90°)} \Rightarrow R = \frac{5k}{2}$$

$$r = \frac{\Delta}{s} = \frac{1}{2}\frac{(3k)(4k)}{6k} = k$$

$$\frac{R}{r} = \frac{5k/2}{k} = \frac{5}{2} = 2.5$$

4. Let f : N $\rightarrow$ R be a function such that f(x + y) = 2f(x) f(y) for natural numbers x and y. If f(1) = 2, then the value of α for which $\sum_{k=1}^{10} f(\alpha + k) = \frac{512}{3}(2^{20} - 1)$ holds, is :

(A) 2 (B) 3 (C) 4 (D) 6

Ans. (C)

Sol. $\because$ f(x + y) = 2f(x).f(y)

Let f(x) 2^{2x-1}

Now $\sum_{k=1}^{10} f(\alpha + k) = \sum_{k=1}^{10} 2^{2(\alpha+k)-1} = 2^{2\alpha-1}.\sum_{k=1}^{10} 2^{2k}$

$$2^{2\alpha-1}.2^2\frac{\left[\left(2^2\right)^{10} - 1\right]}{2^2 - 1} = \frac{2^{2\alpha+1}\left[2^{20} - 1\right]}{3}$$

$2^{2\alpha+1}$ = 512 $\Rightarrow$2α + 1 =9

α = 4

5. Let A be s 3 $\times$ 3 real matrix such that

$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix};A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ and $A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$.

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has :

(A) no solution (B) infinitely many solutions

(C) unique solution (D) exactly two solutions

Ans. (B)

6. Let $f : R \to R$ be defined as

$$f(x) = x^3 + x - 5$$

If g(x) is a function such that $f(g(x)) = x, \forall$ 'x' $\in R$ then g'(63) is equal to _____.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$ (C) $\frac{43}{49}$ (D) $\frac{19}{49}$

Ans. (A)

Sol. $g'(f(x)) = \frac{1}{f'(x)}$ …….. (1)

to find g' (63) put f(x) = 63

$x^3 + x - 5 = 63$

$x(x^2 + 1) = 68 \Rightarrow x = 4$

Now $f'(x) = 3x^2 = 1 \Rightarrow f'(4) = 49$

then the value of $g'(63) = \frac{1}{49}$

7. Consider the following two proposition:

P1 : $\sim(p \to \sim q)$

P2 : $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then :

(A) P1 is TRUE and P2 is FALSE (B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE (D) Both P1 and P2 are TRUE

Ans. (C)

8. If $\frac{1}{2\cdot3^{10}}+\frac{1}{2^2\cdot3^9}+...+\frac{1}{2^{10}\cdot3}=\frac{K}{2^{10}\cdot3^{10}}$, then the remainder when K is divided by 6 is:

(A) 1 (B) 2 (C) 3 (D) 5

Ans. (D)

Sol. $\frac{1}{2^1\cdot3^{10}}+\frac{1}{2^2\cdot3^9}+.....\frac{1}{2^9\cdot3^2}+\frac{1}{2^{10}\cdot3^1}=\frac{K}{2^{10}\cdot3^{10}}$

$\Rightarrow k = 2^9 + 2^8.3 + 2^7.3^2 + \ldots\ldots\ldots\ldots + 2.3^8 + 3^9 = 3^{10} - 2^{10} = (3^5 - 2^5)(3^5 - 2^5)$

$= 211 \times 275$

$= (210 + 1)(270 + 5)$

$= (6\lambda + 1)(6\mu + 5)$

Remainer = 5

9. Let f(x) be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim\limits_{x\to} \frac{f(x)}{x-1}$ is equal to :

(A) – 15 (B) – 60 (C) 60 (D) 15

Ans. (A)

Sol. As $f(x) + f'(x) + f'(x) = x^5 + 64$

$\Rightarrow$ f(x) is a polynomial in x with degree 5

Let $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$

$\Rightarrow f'(x) = 5x^4 + 4ax^3 + 3bx^2 + 2cx + d$

& $f'(x) = 20x^3 + 12ax^2 + 6bx + 2c$

Since $f(x) + f'(x) + f'(x) + f''(x) = x^5 + 64$

$\Rightarrow a+5 = 0,\ b + 4a + 20 = 0\ ;\ c + 3b + 12a = 0,\ d + 2c + 6b = 0$ & $e + d + 2c = 64$

$\Rightarrow a = -5,\ b = 0,\ c = 60,\ d = -120,\ e = 64$

$\Rightarrow f(x) = x^5 - 5x^4 + 60x^2 - 120x + 64$

$\Rightarrow f(x) = (x-1)(x^4 - 4x^3 - 4x^2 + 56x - 64)$

$\Rightarrow \frac{f(x)}{x-1} = x^4 - 4x^3 - 4x^2 + 56x - 64$

$\lim\limits_{x\to} \frac{f(x)}{x-1} = 1 - 4 - 4 + 56 - 64 = -15$

10. Let E_1 and E_2 be two events such that the condition $P(E_1 \mid E_2) = \frac{1}{2}$, $P(E_1 \mid E_2) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then:

(A) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$ (B) $P(E'_1 \cap E'_2) = P(E'_1) \cdot P(E'_2)$

(C) $P(E_1 \cap E'_2) = P(E_1) \cdot P(E_2)$ (D) $P(E'_1 \cap E_2) = P(E_1) \cdot P(E_2)$

Ans. (C)

Sol. $P(E1 \cap E2) = \frac{1}{8} = P(E1)\, P\left(\frac{E_2}{E_1}\right) = P(E2)\, P\left(\frac{E_1}{E_2}\right)$

So $\frac{1}{8} = P(E1) \times \frac{3}{4}$

$P(E1) = \frac{1}{6}$

And $\frac{1}{8} = P(E_2) \times \frac{1}{2}$

$P(E_1) = \frac{1}{4}$

Now option

(1) $P(E_1)\,P(E_2)\ \frac{1}{6}\times\frac{1}{4}=\frac{1}{24} \neq P(E_1 \cap E_2)$

(2) $P(\bar{E}_1).P(\bar{E}_2) = \frac{5}{6}\times\frac{3}{4}=\frac{15}{24}$

$P(\bar{E}_1 \cap \bar{E}_2) = P(\overline{E_1 \cup E_2}) = 1 - p(E_1 \cup E_2)$

$= 1 - [P(E1) + P(E2) - (E_1 \cap E_2)] = 1 - \left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right) = 1-\frac{7}{24}=\frac{17}{24}$

$P(\bar{E}_1 \cap \bar{E}_2) \neq P(\bar{E}_1)P(\bar{E}_2)$

(3) $P(\bar{E}_1 \cap E_2) = P(E_2) - P(E_1 \cap E_2)$

$\frac{1}{4}-\frac{1}{8}=\frac{1}{8} \neq P(E_1)P(E_2)$

(4) $P(E_1 \cap \bar{E}_2) = P(E_1) - P(E_1 \cap E_2)$

$\frac{1}{4}-\frac{1}{8}=\frac{1}{8} \neq P(E_1)P(E_2)$

11. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :

(A) a non-identify symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identify matrix

Ans. (A)

Sol. $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = 4I.$ (symmetric)

& $A^3 = -4A$ (skew symmetric)

$\Rightarrow M = \sum_{k=1}^{10} A^{2k} = [(-4) + (-4)^2 + (-4)^3 + \ldots + (-4)^{10}]\, I$

$= -4\lambda I$ is symmetric

$\Rightarrow N = \sum_{k=1}^{10} A^{2k-1} = A\,[1 + (-4) + (-4)^3 + \ldots + (-4)^9]\, I$

$= \lambda A$ is skew symmetric

Where $\lambda = \{1 + (-4) + (-4)^3 + ... + (-4)^9\}$

Now $MN = -4\lambda^2 A = NM$

$\Rightarrow MN^2 = (MN)N = (NM)N = N(MN) = N(NM) = N^2M$

Hence $(MN^2)^T = (N^2)^T M^T = (N^T)^2 M^T = (-N)^2 M = N^2M$

$\Rightarrow MN^2$ is symmetric matrix

12. Let $g : (0,\infty) \to R$ be a differentiable function such that

$$\int\left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)\left(e^x + 1 - xe^x\right)}{(e^x + 1)^2}\right)dx = \frac{xg(x)}{e^x + 1} + c, \text{ for all } x > 0, \text{ where c is an arbitrary constant.}$$

Then:

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$ (B) g is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0,\frac{\pi}{2}\right)$ (D) g – g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Ans. (D)

13. Let $f : R \to R$ and $g : R \to R$ be two function defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x) = \frac{1 - 2e^{2x}}{e^x}$.

Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha - 1)^2}{3}\right)\right) > f\left(g\left(\alpha\frac{5}{3}\right)\right) \text{ holds?}$$

(A) (2, 3) (B) (–2, –1) (C) (1, 2) (D) (–1, 1)

Ans. (A)

14. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}a_i > 0$, i = 1, 2, 3 be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}$, $\vec{b}$ and x-axis are coplanar, then projection fo a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to:

(A) $\sqrt{7}$ (B) $\sqrt{2}$ (C) 2 (D) 7

Ans. (B)

15. Let y = y(x) be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$.

Then, the point $x = -\frac{4}{3}$ for the curve y = y(x) is :

(A) not a critical point (B) a point of local minima

(C) a point of local maxima (D) a point of inflection

Ans. (B)

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to :

(A) $3 + 4\sqrt{2}$ (B) $-5 + 6\sqrt{2}$ (C) $-4 + 3\sqrt{2}$ (D) $7 + 6\sqrt{2}$

Ans. (C)

Sol. Let equation of tangent $y = mx + \frac{1}{4m}$

then is also tangent to curve

$$\left|\frac{0 - 0 + \frac{1}{4m}}{\sqrt{m^2 + 1}}\right| = \sqrt{2}$$

$$\frac{1}{16m^2} = 2(m^2 + 1)$$

$$32m^4 + 32m^2 - 1 = 0$$

$$m^2 = \frac{-32 + \sqrt{(32)^2 + 4 \times 32}}{64}$$

then the value of $8.\ |m_1.\ m_2| = 8m^2$

$$= 8 \times \left(\frac{-32 + \sqrt{(32)^2 + 4 \times 32}}{64}\right)$$

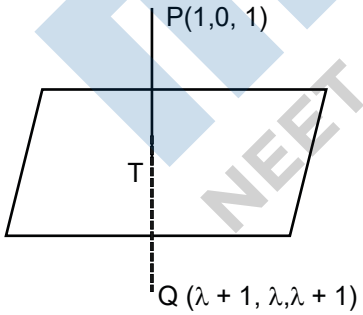
$$= -4 + \sqrt{18} = 3\sqrt{2} - 4$$

17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plans S : x + y + z = 5. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5 (C) 7 (D) 11

Ans. (B)

Sol.



PQ : $\frac{x-1}{1} = \frac{y}{1} = \frac{z-1}{1} = \lambda$

$$\Rightarrow \left(\frac{\lambda+2}{2}, \frac{\lambda}{2}, \frac{\lambda+2}{2}\right) \Rightarrow \frac{3\lambda+4}{2} = 5$$

$\Rightarrow \lambda = 2$ so T (2, 1, 2)

and Q (3, 2, 3)

Let A(1, –1, –1)

AR: $\frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}=t$

$\Rightarrow$ R (1 + t, – 1 + t, – 1 + t)

$\Rightarrow$ – 1 + 3t = 5 $\Rightarrow$ t = 2

So R (3, 1, 1)

$QR^2 = 1 + 4 = 5$

18. If the solution curve y = y(x) of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to:

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Ans. (C)

19. Let $x = 2t$, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis if the conic such that SA $\perp$ BA, where A is any point on the conic. If k is the ordinate of the centroid of the ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to :

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$ (C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Ans. (D)

20. Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to:

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)-\pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right)-\pi$ (C) $\tan^{-1}(3)-\pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

Ans. (B)

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$.

If for $\alpha, \beta \in R$, $C_1 + 3 \cdot 2\,C_2 + 5 \cdot 3\,C_3 + \ldots$ up to 10 terms $= \frac{\alpha \times 2^{11}}{2^\beta - 1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots \text{upto 10 terms}\right)$ then the value of $\alpha + \beta$ is equal to _____.

Ans. (Bonus)

22. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is _____.

Ans. (63)

Sol. Largest digit in number = 9

Sum of three digit maximum can be = 27

Hence sum of digit can be = 7, 14, 21

Now $\boxed{a_1}\boxed{a_2}\boxed{a_3}$ $a_1 + a_2 + a_3 = 7, 14, 21$

C-1 $a_1 + a_2 + a_3 = 7$ a_3 = odd ⟵ C

$a_3 = 1$ $(a_1, a_2) \equiv (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6,0)$ Total number = 6

$a_3 = 3$ $(a_1, a_2) \equiv (1, 3), (2, 2), (3, 1), (4,0)$ Total number = 4

$a_3 = 5$ $(a_1, a_2) \equiv (1, 1), (2,0)$ Total number = 2

C-2 $a_1 + a_2 + a_3 = 14$

$a_3 = 1$ $(a_1, a_2) \equiv (4, 9), (5, 8), ...(9, 4)$ Total number = 6

$a_3 = 3$ $(a_1, a_2) \equiv (2, 9), (3, 8), ...(9, 2)$ Total number = 8

$a_3 = 5$ $(a_1, a_2) \equiv (1, 8), ...(8, 1), (9,0)$ Total number = 9

$a_3 = 7$ $(a_1, a_2) \equiv (1, 6), ...(6, 1), (7,0)$ Total number = 7

$a_3 = 9$ $(a_1, a_2) \equiv (1, 4), ...(4, 1), (5,0)$ Total number = 5

C-3 $a_3 + a_2 + a_3 = 21$

$a_3 = 3$ $(a_1, a_2) \equiv (9, 9)$ Total number = 1

$a_3 = 5$ $(a_1, a_2) \equiv (7, 9), (8, 8), (9, 7)$ Total number = 3

$a_3 = 7$ $(a_1, a_2) \equiv (5, 9), ...(9, 5)$ Total number = 5

$a_3 = 9$ $(a_1, a_2) \equiv (3, 9), ...(9, 3)$ Total number = 7

Total = 63

23. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4|\vec{b}| = 3$ and $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to _____.

Ans. (576)

Sol. Given $|\vec{a}| = 4|\vec{b}| = 3, \vec{a} \wedge \vec{b} \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})^2 + 4(\vec{a} \cdot \vec{b})^2$

$(-\vec{a} \times \vec{b} + \vec{b} \times \vec{a})^2 + 4(\vec{a} \cdot \vec{b})^2$

$= 4|\vec{a}|^2 |\vec{b}|^2$

$= 4 \times 16 \times 9 = 64 \times 9 = 576$

24. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to _____.

Ans. (7)

25. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\text{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is _____.

Ans. (4)

26. For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is _____.

Ans. (4)

Sol. $\Rightarrow \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$

$\Rightarrow \frac{(19^9(12) - 12^9(19)}{57(19^8 - 12^8)}$

$\Rightarrow \frac{19 \times 12(19^8 - 12^8)}{57(19^8 - 12^8)}$

$\Rightarrow 4$

27. Let $f : R \to R$ be a function defined be $f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)\left(2 + x^{25}\right)\right)^{\frac{1}{50}}$. If the function(x) = f(f(f(x))) + f(f(x)), then the greatest integer less then or equal g(1) is _____.

Ans. (2)

Sol. g(x) = f(f(f(x))) + f(f(x))

G(1) = f(f(f(x))) + f(f(x)) .....(1)

$f(1) = (\sqrt{3})$

$f(f(1)) = f(\sqrt{3}) = ((2 - x)(2 + x))^{1/2}$

$= (4 - 3)^{1/2} = 1$

$f(f(f(1))) = f(1) = \sqrt{3}$

$\Rightarrow [G(1)] = [\sqrt{3} + 1] = 2$

28. Let the lines

$L_1 : \vec{r} = \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right), \lambda \in P,$

$L_2 : \vec{r} = \left(\hat{i} + 3\hat{j} + \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + 5\hat{k}\right); \mu \in P,$

Intersect at the point S. If a plane ax + by – z + d = 0 passed through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to _____.

Ans. (5)

29. Let A be 3×3 matrix having entries from the set {– 1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is _____.

Ans. (414)

Sol. Largest digit in number = 9

Sum of three digit maximum can be = 27

Hence sum of digit can be = 7, 14, 21

Now $\boxed{a_1}\boxed{a_2}\boxed{a_3}$ $a_1 + a_2 + a_3 = 7, 14, 21$

C-1 $a_1 + a_2 + a_3 = 7$ a_3 = odd ⟵ C

$a_3 = 1$ $(a_1, a_2) \equiv (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (5,0)$ Total number = 5

$a_3 = 3$ $(a_1, a_2) \equiv (1, 3), (2, 2), (3, 1),(3,0)$ Total number = 3

$a_3 = 5$ $(a_1, a_2) \equiv (1, 1)$ Total number = 1

C-2 $a_1 + a_2 + a_3 = 14$

$a_3 = 1$ $(a_1, a_2) \equiv (4, 9), (5, 8), ...(9, 4)$ Total number = 6

$a_3 = 3$ $(a_1, a_2) \equiv (2, 9), (3, 8), ...(9, 2)$ Total number = 8

$a_3 = 5$ $(a_1, a_2) \equiv (1, 8), ...(8, 1)$ Total number = 8

$a_3 = 7$ $(a_1, a_2) \equiv (1, 6), ...(6, 1)$ Total number = 6

$a_3 = 9$ $(a_1, a_2) \equiv (1, 4), ...(4, 1)$ Total number = 4

C-3 $a_1 + a_2 + a_3 = 21$

$a_3 = 3$ $(a_1, a_2) \equiv (9, 9)$ Total number = 1

$a_3 = 5$ $(a_1, a_2) \equiv (7, 9), (8, 8), (9, 7)$ Total number = 3

$a_3 = 7$ $(a_1, a_2) \equiv (5, 9), ...(9, 5)$ Total number = 5

$a_3 = 9$ $(a_1, a_2) \equiv (3, 9), ...(9, 3)$ Total number = 7

Total = 57

30. The greatest integer less then or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \ldots$ is equal to _____.

Ans. (98)

Sol. $S = \left(1-\frac{2}{3}\right) + \left(1-\frac{4}{9}\right) + \left(1-\frac{8}{27}\right)$ + upto 100 terms

$$S = 100 - \left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ...100\text{term}\right)$$

$$S = 100 - \frac{2}{3}\left(\frac{1-\left(\frac{2}{3}\right)^{100}}{1-\frac{2}{3}}\right)$$

$$S = 100 - 2\left(1-\left(\frac{2}{3}\right)^{100}\right)$$

$$S = 98 + 2\left(\frac{2}{3}\right)^{100}$$

$$0 < 2\times\left(\frac{2}{3}\right)^{100} < 1$$

[s] = 98